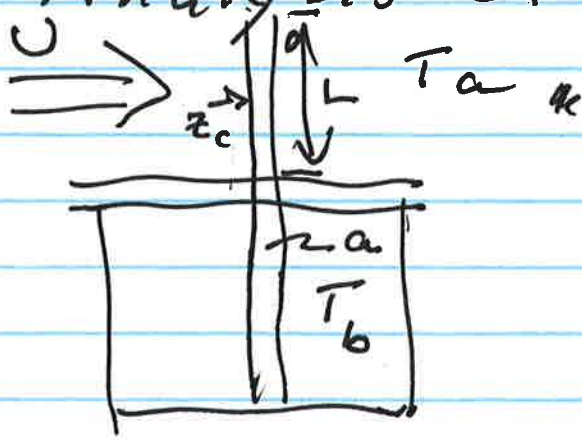
Analysis of Demo



T_c ≡ temp at which color changes

z_c ≡ location of color change

we define:

$$z^* = z/L \, , \quad T^* = \frac{T - T_a}{T_b - T_a}$$

Assuming that $T\big|_{r=a} \approx \bar{T}$ for all z,

$$\frac{\partial^2 \bar{T}^*}{\partial z^{*2}} - \left[\frac{2hL^2}{ka}\right] T^*\Big|_{r^*=1}$$

$$\hookrightarrow \approx \bar{T}^*$$

w/ BCs $\bar{T}^*\Big|_{z^*=0} = 1 \quad \dfrac{\partial \bar{T}^*}{\partial z^*}\Big|_{z^*=0} = 0$

$$\therefore \bar{T}^* = \cosh \lambda z^* - \frac{\sinh \lambda}{\cosh \lambda} \sinh \lambda z^*$$

$$\lambda \equiv \left[\frac{2hL^2}{ak}\right]^{1/2}$$

We need to determine λ!

For demo, $2a = \frac{1}{2}''$ $\therefore a = 0.635\,cm$

$L = 10\,cm$

K is uncertain – depends on Al alloy!

most common is Al-6061 which has the properties:

$$K = 167\ W/m\,°K, \quad \rho = 2.7\ g/cm^3, \quad \hat{C}_p = 0.896\ \frac{J}{g\,°C}$$

$$\therefore \alpha = \frac{K}{\rho \hat{C}_p} = 0.69\ cm^2/s$$

We also need h!

Use Whittaker:

$$h = \frac{K_{air}}{D}\left(0.4\,Re^{1/2} + 0.06\,Re^{2/3}\right) P_r^{0.4} \left(\frac{\mu_\infty}{\mu_0}\right)^{1/4}$$

For air $P_r = 0.71$ and $\frac{\mu_\infty}{\mu_0} \simeq 1$

(not much change for small ΔT)

So we <u>need</u> Re!

Empirically fond that rod changes color half-way up w/ dryer about 17" away, air flow ≈ 8 m/s

$\nu_{air} = 0.15\ cm^2/s \qquad k_{air} = 0.026\ \frac{W}{m\,^oK}$

$$Re = \frac{UD}{\nu} = \frac{(800)(1.27)}{(0.15)} = 6.8 \times 10^3$$

This yields:

$$h = \frac{k}{D} \times 47.5 = \frac{0.026}{0.0127} \times 47.5 = 97.3\ \frac{W}{m\,^oK}$$

Note that this varies roughly as $U^{1/2}$ so not-too-sensitive...

What's $\lambda = \left[\frac{2hL^2}{ak}\right]^{1/2}$?

$$= \left[\frac{2 \times 97.3 \times (0.1)^2}{0.0635 \times 167}\right]^{1/2} = 1.35$$

Temperature Profile of a Cooled Rod

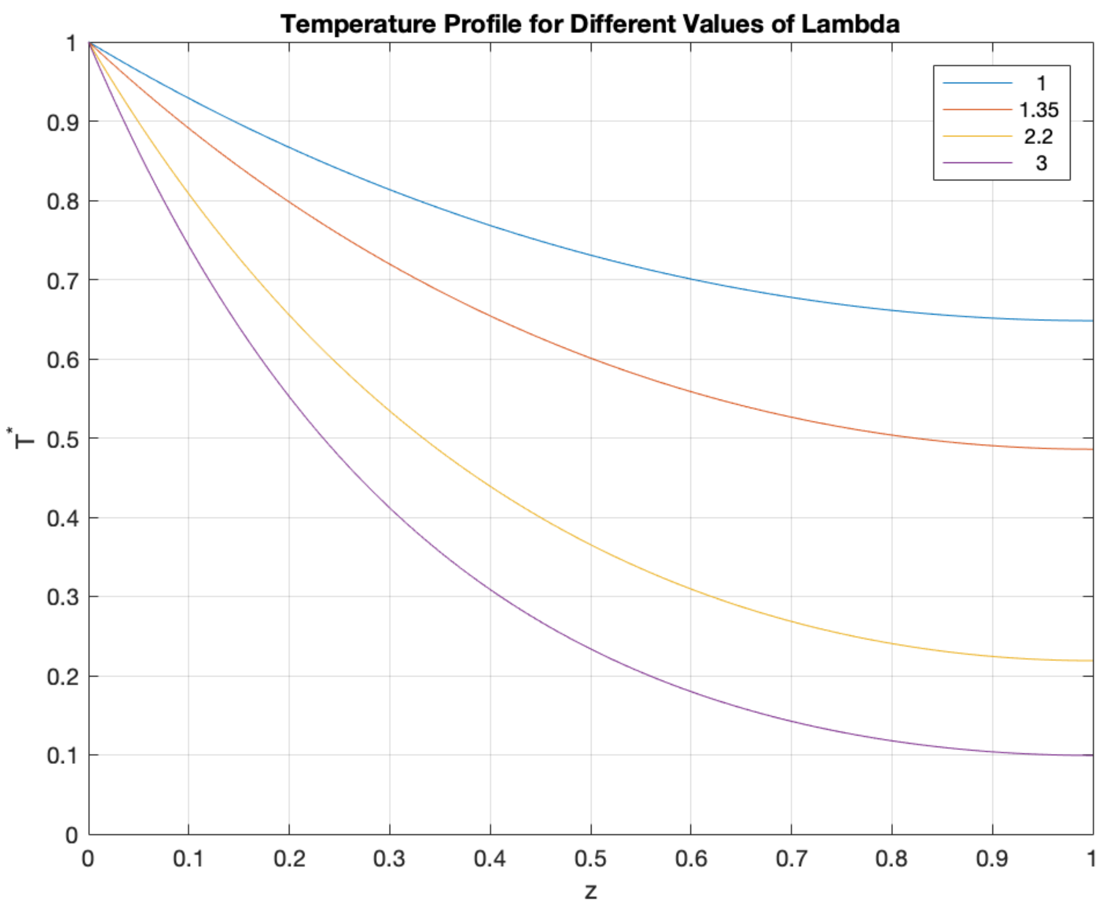
We have the dimensionless temperature profile, which is a function of lambda. We are interested in the profile for values of lambda ranging from 1 to 3.

```
z = [0:.01:1];

la = [1,1.35,2.2,3]';

T = cosh(la*z) - ((sinh(la)./cosh(la))*ones(size(z))).*sinh(la*z);

figure(1)
plot(z,T)
grid on
xlabel('z')
ylabel('T^*')
title('Temperature Profile for Different Values of Lambda')
legend(str2mat(num2str(la)))
```



For our experiment we got $z^* = \frac{1}{2}$ where $T_b = 6.5°C$, $T_{air} = 38.3°C$ and transition is at $T_c = 26.5°C$

$$\therefore \quad T_c^* = \frac{26.5 - 38.3}{6.5 - 38.3} = 0.37$$

From our solution, however, T^* at $z^* = 1$ is 0.49 - it never gets down to this value! At $z^* = \frac{1}{2}$ $T^* = 0.6$ for this value of λ.

<u>If</u> $\lambda = 2.2$, however, it would match. Also, if $T_b = 18.5°C$ we would match at $\lambda = 1.35$.

Sources of error:

1) underestimate h – by using too low a velocity.

$$\lambda \sim h^{1/2}, \quad h \sim U^{1/2} \therefore \lambda \sim U^{1/4}$$

we would need to increase λ by a factor $\frac{2.2}{1.35} = 1.63$ so this would require U higher by ~ 7x! unlikely.

2) overestimate k (use value for wrong alloy)

$\lambda \sim k^{-1/2}$ so would require

$$k = \frac{167\ W/m^{\circ}K}{(1.63)^2} = 63\ W/m^{\circ}K$$

This is possible as alloy conductivities range from 70 to 236 $W/m^{\circ}K$

– would have to measure another way! Still unlikely...

3) we may violate assumption that

$$T^*\Big|_{r^*=1} = \bar{T}^*$$

We can <u>estimate</u> this effect

Recall from last lecture:

$$T = T\Big|_{r=a} + \frac{\dot{S}a^2}{K}\frac{1}{4}\left(1 - \frac{r^2}{a^2}\right)$$

<u>and</u> $\bar{T} = T\Big|_{r=a} + \frac{1}{8}\frac{\dot{S}a^2}{K}$

Now $q_r\Big|_{r=a} = \frac{1}{2}\dot{S}a = h\left(T\Big|_{r=a} - T_a\right)$

$$\therefore \bar{T} = T\Big|_{r=a} + \frac{1}{4}\frac{ha}{K}\left(T\Big|_{r=a} - T_a\right)$$

$$\frac{ha}{K} = \frac{(97.3)(0.00635)}{167} = 0.0037$$

so $\bar{T} = T\Big|_{r=a} + 0.0009\left(T\Big|_{r=a} - T_a\right)$

which is a <u>negligible</u> <u>correction</u>

4) The temp. of the rod may not match the temp. of the bath!

Our bath water was unstirred - so heat transfer is via natural convection. We have for a vertical surface:

$$h_b \cong \frac{K}{L_b} \cdot 0.61 Ra^{1/4}$$

→ for water, dep. on Pr

where Ra ≡ Rayleigh #

$$Ra = \frac{g L_b^3 \beta \Delta T}{\alpha \nu}$$

for water $\beta = 2.2 \times 10^{-4}$ 1/°K, $\alpha = 0.0014$ cm²/s

$\nu = 0.0097$ cm²/s, $g = 980$ cm/s²

Take $\Delta T \sim 5°C$, $L = 5$ cm

∴ $Ra = 9.9 \times 10^6$, $h_b = 419 \frac{W}{m^2 K}$

This is higher than our forced air of 97 $W/m^{o}K$ - but not by that much! In addition the immersed length is shorter (5cm vs. 10cm). We can estimate the temp. at the middle via a heat balance! The energy from both sides must be the same!

From the notes:

$$Q = \frac{k\pi a^2 (T_o - T_a)}{L} \lambda \frac{\sinh\lambda}{\cosh\lambda}$$

and in the bath:

$$Q = \frac{k\pi a^2 (T_b - T_o)}{L_b} \lambda_b \frac{\sinh\lambda_b}{\cosh\lambda_b}$$

Now $\lambda_b = \left[\frac{2 h_b L_b^2}{a k}\right]^{1/2} = 1.406$ vs. $\lambda = 1.35$

equating Q's and solving yields:

$$T_o = \frac{T_a + 2.1 T_b}{1 + 2.1} = 16.8°C!$$

$$\therefore T_c^* = \frac{38.3 - 26.5}{38.3 - 16.8} = 0.55!$$

Thus, while other sources of error may be sig., this looks like the main issue! We could test this by stirring up the bath to increase h_b (by a very large factor!).